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FRACTIONAL ORDER ELECTRICAL IMPEDANCE OF FRUITS AND VEGETABLES

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ABSTRACT
The idea of fractional calculus is not new. Fractional derivatives are almost as old as integer-order definition. In 1695 Leibniz discussed this problem with L'Hospital, but many other contributions are due to investigators such as Liouville, Abel, Heaviside and Riemann, that formalized the theory of the non-integer order systems. The area of fractional calculus has primarily been the domain of mathematicians, and only had the theoretical foundation. Nowadays, this concept is employed in physics, engineering, biology, economy and other scientific fields. In our work, we apply the concepts of fractional calculus and the theory of electrical impedance to botanical elements. The fractional order behaviour of these type of systems are studied and the relation with the electrical impedance is formulated.

KEY WORDS
Fractional calculus, electricity, impedances.

1. Introduction

Fractional calculus (FC) is a generalization of integration and differentiation to non-integer order, being the fundamental operator is ${}_aD_t^{\alpha}$, where α and t are the limits of the operation [1 - 8].

FC was employed to describe several physical phenomena such as heat flow, electricity, magnetism, and fluid dynamics. For example the diffusion problem was solved by introducing the derivative of order $\alpha = 0.5$ in the equation to solve it in order to time [6]. On the other hand, the electromagnetic theory adopted the FC to describe the charge distribution of a dipole [7].

Recent studies have brought FC into attention revealing that many physical phenomena are modeled by fractional differential equations [4 - 5]. The importance of fractional order mathematical models is that it can be used to make a more accurate prediction and to give a deeper insight into the physical processes underlying long range memory behaviors [6].

Bearing these ideas in mind, this paper analyzes the fractional-order dynamics in botanical electrical impedances and is organized as follows. Section 2 introduces the fundamental aspects of the theory of fractional calculus. Section 3 presents the fundamental electrical impedance concepts. Section 4, describes the research work in the field of fractional order electrical impedances. Finally, section 5 draws the main conclusions.

2. Fractional Calculus and Impedance

Since the beginning of the theory of differential and integral calculus, mathematicians such as Euler and Liouville investigated their ideas on the calculation of non-integer order derivatives and integrals [1 - 9]. An important property revealed by the Grünwald-Letnikov definition is that while an integer-order derivative implies simply a finite series, the fractional-order derivative requires an infinite number of terms. Fractional derivatives have implicitly a "memory" of all past events.

The fractional derivative introduced by Riemann and Liouville is a generalization of the classical definition of the derivative. It has been recently use in several applications in the domain of control and identification [6 - 9].

Fractional-order circuits and systems have witnessed an increasing interest lately [10]. Capacitors are one of the crucial elements in integrated circuits and are used extensively in many of them, such as sample and holds, radio-frequency oscillators, mixers [11 - 12]. Jonscher [12] demonstrated that the ideal capacitor cannot exist in nature because, an impedance of the form $1/[(j\omega)C]$ would violate causality [13]. The dielectric material exhibits a realistic fractional behaviour when $1/[(j\omega)^{\alpha}C]$, with $\alpha \approx 0.5$. It is important to mention that in a fractional capacitor of order α the phase shift yields $\alpha\,\pi/2$.

3. On The Electrical Impedance

Commonly, electrical elements consist on resistors, capacitors and inductors. These elements have a basis of physical electrochemical systems [14].

The voltage signal is expressed as a function of time, described by the following equation:

$$u(t) = U_0 \cos(\omega t) \quad (1)$$

where $u(t)$ is the voltage at time t, U_0 is the amplitude of the signal and ω is the frequency. Similarly, for the current we have:

$$i(t) = I_0 \cos(\omega t + \phi) \quad (2)$$

where $i(t)$ is the current at time t, I_0 is the amplitude of signal and ϕ is the phase shift.

The voltage and current, can be expressed as:

$$u(t) = \mathrm{Re}\left\{U_0 e^{j(\omega t)}\right\} \quad (3)$$

$$i(t) = \mathrm{Re}\left\{I_0 e^{j(\omega t + \phi)}\right\} \quad (4)$$

and the impedance Z depends on ω:

$$Z(j\omega) = \frac{U(j\omega)}{I(j\omega)} = Z_0 e^{j\phi} \quad (5)$$

In this last equation, we find a complex number and, consequently, it is possible to obtain its polar plot for different values of ω. For example, Fig. 1 represents the polar diagram for a parallel *RC* circuit, where the low (high) frequencies are located at the right (left) hand side. For an impedance $Z(j\omega)$, resulting from a parallel *RL*, the polar diagram is symmetrical and the frequency varies in the opposite direction.

Electrochemical impedance plots often present either several semicircles or a portion only of a semicircle.

Table I shows the polar plots of impedance $Z(j\omega)$ and admittance $Y(j\omega) = Z^{-1}(j\omega)$ for simple series and parallel association of *RL* and *RC* circuits, where $G = \mathrm{Re}\{Y\}$ is the conductance and $B = \mathrm{Im}\{Y\}$ is the susceptance.

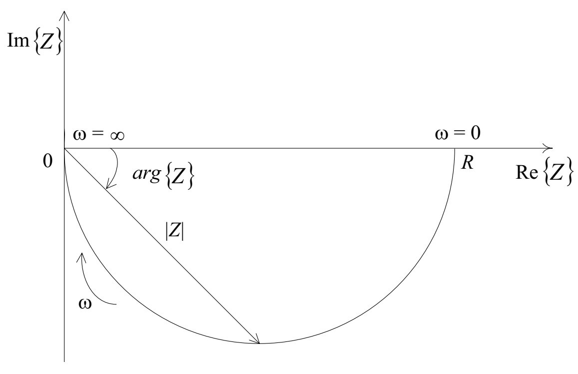
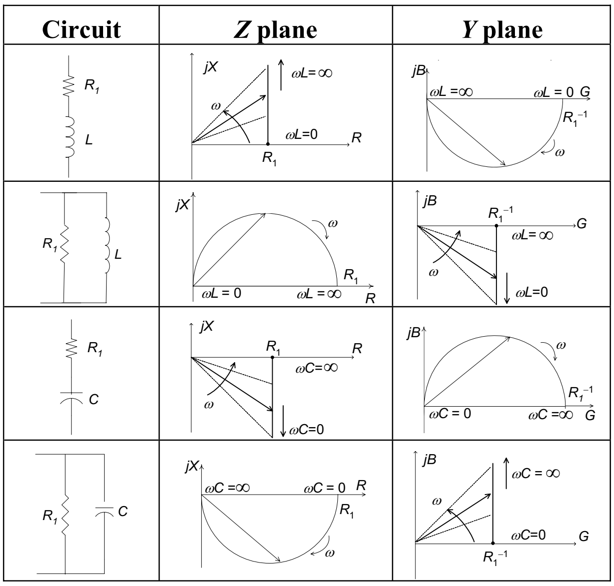


Fig 1. Polar diagram for impedance *Z* (*jω*)

Table I . Impedance *Z* (*jω*) and admittance *Y* (*jω*) *loci* of *RL* and *RC* circuits



4. Study of Fractional Order Electrical Impedances

The structure of fruits and vegetables have cells that are sensitive to heat, pressure and other stimuli. These systems constitute electrical circuits exhibiting a complex behaviour. Bearing these facts in mind, in our work we study the electrical impedance for several botanical elements, under the point of view of fractional order systems.

We apply sinusoidal excitation signals $v(t)$ for several distinct frequencies ω (Fig. 2) and the impedance Z, in the botanical system, is measured based on the resulting voltage u and current i. Moreover, we measure the environmental temperature, the weight, the length and width of all botanical elements. This criterion helps us to understand how these factors influence $Z(j\omega)$.

In this study we develop several different experiments for evaluating the variation of the impedance with the amplitude of the input signal V_0, for different electrode lengths of penetration inside the element Δ, environmental temperatures T, weights W and dimension D.

The value of R is changed for each case in order to adapt the values of the voltage and current to the scale of the measurement device.

We start by analyzing the impedance for an amplitude of input signal of $V_0 = 10$ volt, applied to one *Solanum Tuberosum* (potato), with an weight $W = 1.24\ 10^{-1}$ kg, dimension $D = 7.97\ 10^{-2} \times 5.99\ 10^{-2}$ m, environmental temperature $T = 26.5$ degree Celsius, and the electrode length penetration $\Delta = 2.1\ 10^{-2}$ m.

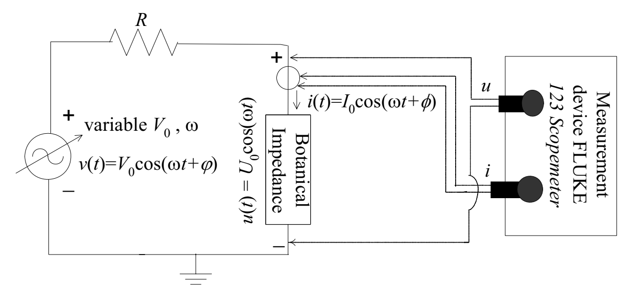


Fig 2. Electrical circuit for the measurement of the botanical impedance $Z(j\omega)$

Figure 3 presents the Bode diagrams for $Z(j\omega)$ and Fig. 4 the corresponding polar plot. The results reveal that the system has a fractional order impedance. In fact, approximating the experimental results in the amplitude Bode diagram through a power function namely by $|Z(j\omega)| = a\omega^{-b}$, we obtain $(a, b) = (4.91\ 10^3, 0.0598)$, at the low frequencies and $(a, b) = (7.94\ 10^5, 0.5565)$, at the high frequencies.

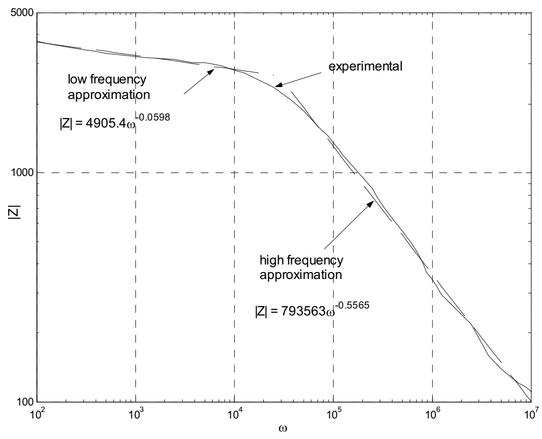
It is interesting to compare the polar diagram of Fig. 4 and the admittance loci presented in Table I. We verify that our systems have similarities with the *RC* series circuit and, therefore, we conclude that this vegetable has proprieties similar to capacitor.

In order to analyze the system linearity we evaluate $Z(j\omega)$ for different amplitudes of input systems, namely, $V_0 = \{5, 15, 20\}$ volt. The impedance has a fractional order and this characteristic does not change significantly with the variation of input signal amplitude (Table II). Therefore, we can conclude that this system has a linear characteristic.

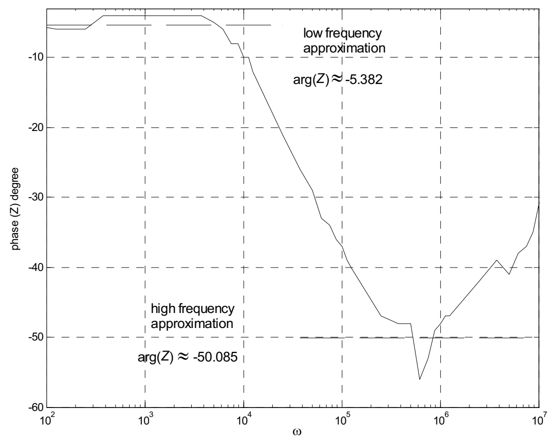
In a second experiment, we vary the length Δ of the electrode penetration inside the potato, and we evaluate its influence upon the value of the impedance. Therefore, we adjust the electrode to $\Delta = 1.42\ 10^{-2}$ m, with $V_0 = 10$ volt, leading to $Z(j\omega)$ approximations $(a, b) = (5.48\ 10^3, 0.0450)$ at the low frequencies, and $(a, b) = (1.00\ 10^6, 0.5651)$ at the high frequencies. With these results, we conclude that the length of wire inside the potato does not change significantly the values of the fractional orders and the linearity is confirmed.

The last experiment with the potato is related with the variation of environmental temperature. In this case, we use the first potato and the same conditions of first experience, but with an temperature $T = 25.7$ degree Celsius. The impedance has the values: $(a, b) = (8.91\ 10^3, 0.0555)$ at the low frequencies and $(a, b) = (7.10\ 10^5, 0.5010)$ at the high frequencies. Once more we verify the small variation of the fractional order.

Another issue that may influence the results is the weight. Therefore, we apply an input signal with amplitude $V_0 = 10$ volt, with environmental temperature $T = 26.5$ degree Celsius, and electrode penetration $\Delta = 2.1\ 10^{-2}$ m



a)



b)

Fig 3. Bode diagrams of the impedance $Z(j\omega)$ for the potato a) amplitude b) phase.

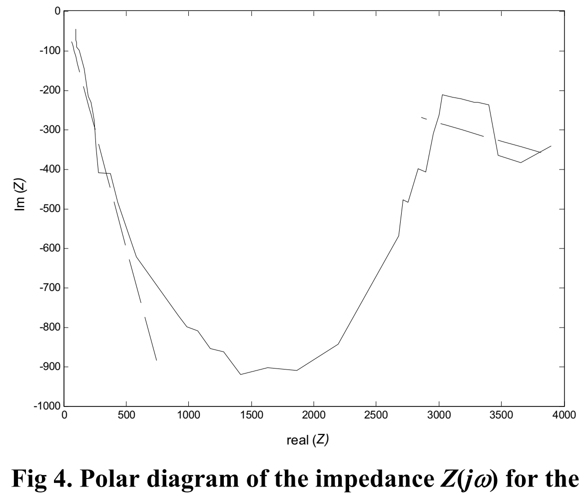


Fig 4. Polar diagram of the impedance $Z(j\omega)$ for the potato

to another potato with weight $W = 5.89\ 10^{-2}$ kg, dimension $D = 7.16\ 10^{-2} \times 3.99\ 10^{-2}$ m. The asymptotic results for $|Z(j\omega)|$ are $(a, b) = (7.17\ 10^3, 0.0546)$ at the low frequencies and $(a, b) = (2.00\ 10^6, 0.5990)$ at the high frequencies. Again, this experience does not reveal significant variations in the fractional order and the linearity is confirmed.

Table II . Comparison the values of $|Z(j\omega)| \approx a\omega^{-b}$ for different amplitudes of the input signal

Amplitude (volt)	low ω		high ω	
	a	*b*	*a*	*b*
5	$4.79\ 10^3$	0.062	$6.52\ 10^5$	0.542
10	$4.91\ 10^3$	0.060	$7.94\ 10^5$	0.557
15	$4.54\ 10^3$	0.054	$5.66\ 10^5$	0.530
20	$4.65\ 10^3$	0.055	$5.86\ 10^5$	0.530

Table III . Characteristics of the vegetables

Vegetable / Specie	Weight (kg)	Length (m)	Width (m)
Carrot / *Daucus Carota L.*	$8.85\ 10^{-2}$	$1.55\ 10^{-1}$	$3.39\ 10^{-2}$
Garlic / Allium sativum *L.*	$2.99\ 10^{-3}$	$1.38\ 10^{-2}$	$6.00\ 10^{-3}$
Onion / *Allium cepa L.*	$8.33\ 10^{-2}$	$5.86\ 10^{-2}$	$5.77\ 10^{-2}$
Potato / *Solanum tuberosum*	$1.24\ 10^{-1}$	$7.97\ 10^{-2}$	$5.99\ 10^{-2}$
Pimento / *Capsicum annuum*	$1.30\ 10^{-1}$	$1.23\ 10^{-1}$	$8.20\ 10^{--2}$
Tomato/*Lycopersicom esculentum*	$1.46\ 10^{-1}$	$5.57\ 10^{-2}$	$6.88\ 10^{-2}$
Turnip / *Brassica napobrassica*	$7.90\ 10^{-2}$	$7.26\ 10^{-2}$	$5.43\ 10^{-2}$

Table IV . Comparison the values of $|Z(j\omega)| \approx a\omega^{-b}$ for different vegetables

Vegetable	low ω		high ω	
	a	*b*	*a*	*b*
Carrot	$1.89\ 10^4$	0.021	$5.00\ 10^7$	0.749
Garlic	$1.65\ 10^4$	0.068	$7.00\ 10^6$	0.621
Onion	$8.38\ 10^3$	0.093	$2.30\ 10^5$	0.463
Potato	$4.91\ 10^3$	0.060	$7.94\ 10^5$	0.557
Pimento	$2.25\ 10^4$	0.041	$2.00\ 10^6$	0.539
Tomato	$2.88\ 10^2$	0.011	$8.68\ 10^3$	0.334
Turnip	$3.92\ 10^3$	0.040	$2.00\ 10^6$	0.581

Table V . Characteristics of the fruits

Fruit / Specie	Weight (kg)	Length (m)	Width (m)
Apple / *Malus domestica*	$1.39\ 10^{-1}$	$6.36\ 10^{-2}$	$7.15\ 10^{-2}$
Banana / *Musa ingens*	$1.11\ 10^{-1}$	$1.49\ 10^{-1}$	$3.42\ 10^{-2}$
Kiwi / *Actinidia deliciosa*	$8.95\ 10^{-2}$	$6.52\ 10^{-2}$	$5.50\ 10^{-2}$
Lemon / *Citrus × limon*	$1.66\ 10^{-1}$	$9.19\ 10^{-2}$	$6.58\ 10^{-2}$
Orange / *Citrus sinensis*	$1.53\ 10^{-1}$	$6.69\ 10^{-2}$	$6.98\ 10^{-2}0$
Pear / *Pyrus communis*	$9.72\ 10^{-2}$	$6.51\ 10^{-2}$	$5.63\ 10^{-2}$

Table VI . Comparison the values of $|Z(j\omega)| \approx a\omega^{-b}$ for different fruits

Fruit	low ω		high ω	
	a	*b*	*a*	*b*
Apple	$7.55\ 10^3$	0.029	$2.00\ 10^6$	0.571
Banana	$3.03\ 10^4$	0.036	$2.00\ 10^7$	0.694
Kiwi	$2.97\ 10^2$	0.018	$5.16\ 10^3$	0.291
Lemon	$1.63\ 10^3$	0.057	$3.33\ 10^5$	0.569
Orange	$1.86\ 10^4$	0.104	$1.00\ 10^6$	0.539
Pear	$4.74\ 10^2$	0.009	$2.04\ 10^4$	0.349

In conclusion, the impedance does not change significantly with the factors analyzed. In this line of thought, we organize similar experiments with other vegetables and fruits.

The results correspond to experiments adopting an amplitude of input signal $V_0 = 10$ volt and an electrode penetration $\Delta = 2.1\ 10^{-2}$ m. Tables III and IV present the characteristics of the vegetables and the corresponding approximation values, respectively.

Similar experiments are developed for several fruits. Tables V and VI present their characteristics and the parameters of the approximation, respectively.

Figures 5 and 6 depict $\mathrm{Re}\{Z(j\omega)\}$ and $\mathrm{Im}\{Z(j\omega)\}$ for the vegetables and fruits under study. We verify that we have distinct low and high frequency responses. For the vegetables carrot, garlic, onion, potato and turnip we get Table VII and for the fruits apple, banana, kiwi and lemon, we get Table VIII.

The results reveal that $Z(j\omega)$ has distinct characteristics according with the frequency range. For low frequencies, the impedance is approximately constant, but for high frequencies it is clearly of fractional order.

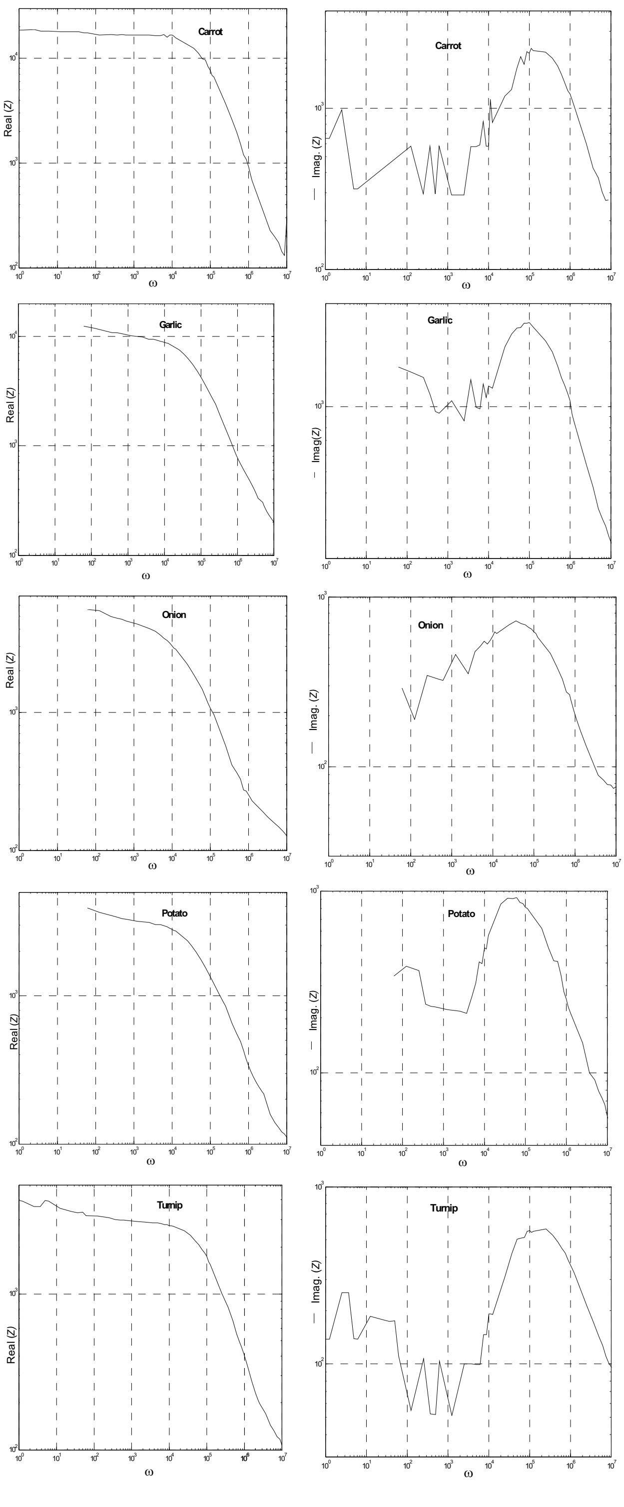


Fig 5. Diagrams of real and imaginary parts of the impedance $Z\,(j\omega)$ for several vegetables: carrot, garlic, onion, potato and turnip.

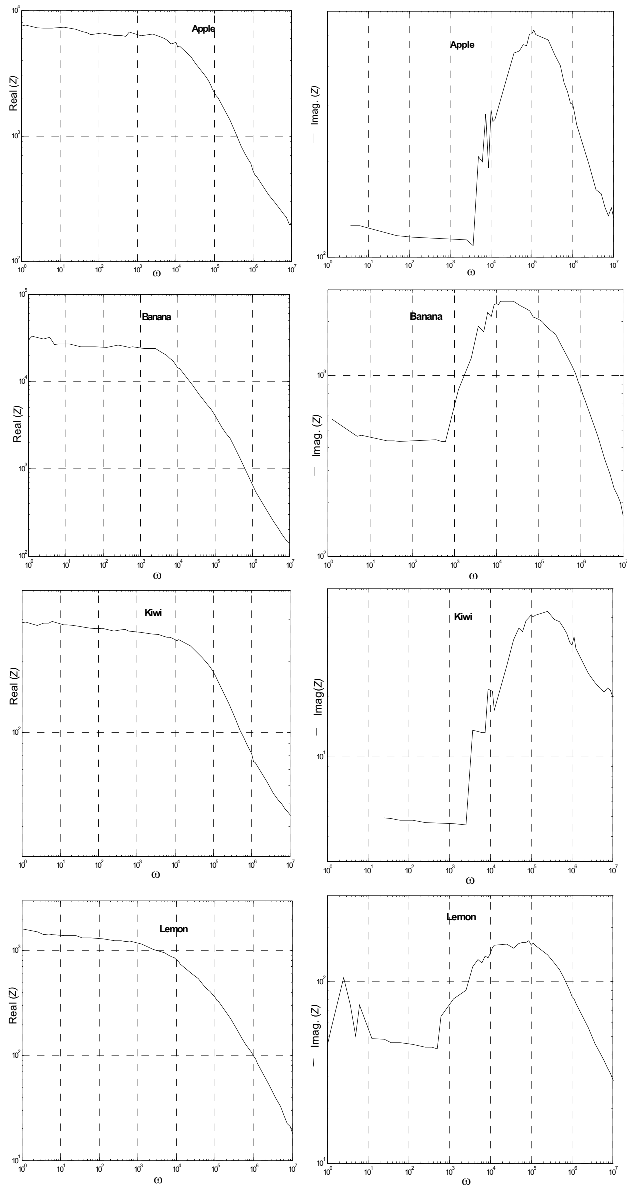


Fig 6. Diagrams of real and imaginary parts of the impedance $Z\,(j\omega)$ for several fruits: apple, banana, kiwi and lemon.

Recent research focus on the implementation of fractional – order capacitances, often called fractances. Patents and commercial products are presently available, opening promising areas of application in electronics and control [15]. This article follows an alternative strategy, studying natural living systems instead of technological artificial elements. Consequently, it points out interesting new directions towards the design of devices capable of measuring how mature is the fruit and vegetable, or to give an estimative of its life span for storage purposes.

Table VII . Comparison the values of real $Re\{Z\} = a\omega^{-b}$ and imaginary $Im\{Z\} = a\omega^{-b}$ parts for several vegetables: carrot, garlic, onion, potato and turnip.

Vegetable		Re {*Z*}		Im {*Z*}	
		a	*b*	*a*	*b*
Carrot	low ω	$1.85\ 10^4$	0.015	$3.11\ 10^2$	0.124
	high ω	$2.82\ 10^8$	0.915	$1.87\ 10^7$	0.701
Garlic	low ω	$1.61\ 10^4$	0.064	$1.42\ 10^3$	0.028
	high ω	$8.09\ 10^6$	0.662	$5.03\ 10^6$	0.618
Onion	low ω	$10.0\ 10^3$	0.124	$1.12\ 10^2$	0.177
	high ω	$2.47\ 10^5$	0.480	$1.69\ 10^5$	0.484
Potato	low ω	$4.95\ 10^3$	0.062	$1.95\ 10^2$	0.066
	high ω	$9.31\ 10^5$	0.593	$7.01\ 10^5$	0.579
Turnip	low ω	$3.96\ 10^3$	0.043	$1.85\ 10^2$	0.107
	high ω	$1.88\ 10^6$	0.610	$5.26\ 10^5$	0.532

Table VIII . Comparison the values of real $Re\{Z\} = a\omega^{-b}$ and imaginary $Im\{Z\} = a\omega^{-b}$ parts for several fruits: apple, banana, kiwi and lemon.

Fruits		Re {*Z*}		Im {*Z*}	
		a	*b*	*a*	*b*
Apple	low ω	$7.45\ 10^3$	0.025	$1.05\ 10^2$	0.057
	high ω	$1.07\ 10^6$	0.540	$4.12\ 10^4$	0.357
Banana	low ω	$3.01\ 10^4$	0.035	$5.10\ 10^2$	0.031
	high ω	$1.86\ 10^7$	0.737	$5.95\ 10^6$	0.644
Kiwi	low ω	$2.97\ 10^2$	0.018	$1.04\ 10^0$	0.321
	high ω	$5.60\ 10^3$	0.303	$1.51\ 10^3$	0.268
Lemon	low ω	$1.56\ 10^3$	0.039	$5.81\ 10^1$	0.043
	high ω	$1.44\ 10^6$	0.697	$4.14\ 10^4$	0.450

5. Conclusion

Fractional calculus was introduced in science on a pure mathematical viewpoint. Nowadays, it is applied in several physical and industrial fields. This paper studied the fractional electrical impedance for several vegetables and fruits. In this line of thought were developed several experiments for measuring the impedance of botanical elements. Our study was performed in the frequency domain, based in the Bode and polar diagrams. The results reveal that all elements have different characteristics for low and high frequencies; however, the impedance remains linear when the system conditions are modified. Moreover, the impedances have fractional order characteristics for high frequencies and reveal similarities with electrical fractional capacitors, called fratances.
We conclude that fractional calculus is an important tool to describe physical phenomena, adopting different concepts of classical methodologies.

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